



**FURTHER MATHEMATICS  
STANDARD LEVEL  
PAPER 1**

Monday 23 May 2005 (afternoon)

1 hour

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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Given that  $f(x) = \sqrt[3]{1+x}$ , use the Maclaurin series, up to the term in  $x^3$ , to find an approximate value of  $\sqrt[3]{1.2}$ . Give your answer correct to 5 decimal places.
2. Prove by mathematical induction that a tree with  $n$  vertices has exactly  $n-1$  edges, where  $n \in \mathbb{Z}^+$ .
3. Consider the group  $(G, \times)$  with the identity element  $e$ . Given two elements  $a$  and  $b$  of the group such that  $ab^2a = b$  and  $a^2 = b^3 = e$ , show that
  - (a)  $ab = b^2a$ ;
  - (b)  $(ab^2)^2 = e$ .
4. The following table shows the number of females and males who are left handed or right handed.

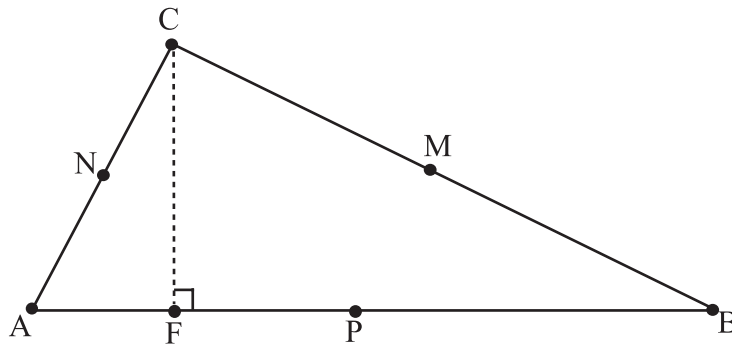
	Left	Right
Female	43	357
Male	76	524

At the 1 % level of significance, is there evidence of an association between the gender of a person and whether they are left or right handed?

5. Find the general solution of the recurrence relation  $a_{n+2} = 5a_{n+1} - 6a_n$ ,  $n \geq 1$ . What are the initial conditions for the sequence  $\{a_n\}$  to generate powers of 3?
6. Determine whether or not the following series is convergent.

$$\frac{1}{\sqrt[3]{2}} + \frac{4}{\sqrt[3]{4}} + \frac{7}{2} + \frac{10}{2\sqrt[3]{2}} + \dots$$

7. In the triangle ABC,  $AC < BC$ , as shown in the following diagram.



The points M, N and P are the midpoints of the sides  $[BC]$ ,  $[CA]$  and  $[AB]$  respectively. F is the foot of the perpendicular from C to  $[AB]$ .

- (a) Show that  $MP = FN$ .
  - (b) Hence or otherwise show that MNFP is a cyclic quadrilateral.
8. The weights of a particular type of nail follow a normal distribution with mean 5.2 g and standard deviation 0.7 g.
- (a) A random sample of 50 nails is taken and the mean weight calculated. Calculate the probability that this sample mean is less than 5 g.
  - (b) A random sample is taken such that the probability of the sample mean exceeding 5.3 g is less than 0.2. Find the minimum sample size.
9. An ellipse is given by the parametric equations  $x = 2 \cos t + 3$  and  $y = 3 \sin t - 1$ . Find the coordinates of the centre and the foci of this ellipse.
10. Let  $V$  be the set of all directed line segments in the plane. The relation  $\cong$  on  $V \times V$  is defined as follows.
- $\vec{AB} \cong \vec{CD}$  if and only if  $[AD]$  and  $[BC]$  have a common midpoint, where  $\vec{AB}$  represents the directed line segment from A to B. Show that  $\cong$  is an equivalence relation.
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